AD717320

AD DDC

TRANS 501

STATSIONARNAYA MODEL' DVIZHENIYA VOD V STRATIFITSIROVANNOM PROLIVE (V PRIMENENII K BOSFORU I NEKOTORYM DRUGIM PROLIVAM)

(Stationary Model of Water Movement in a Stratified Strait (using Bosporus [Istanbul Boǧazi] and Other Straits as Examples))

by

D. M. TOLMAZIN

INSTITUT BIOLOGII YUZHNYKH MOREY

Vol. 20
No. 2
Pp. 18-34

KIEV 1967

Translator: M. Slessers
Editor : W. Deebel
Typed by : B. Davis

U. S. NAVAL OCEANOGRAPHIC OFFICE
WASHINGTON, D. C. 20390
1970

21

ABSTRACT

Two-layer water stratification in straits, such as the Bosporus, Bab-el-Mandeb, and Gibraltar, is discussed. Conclusions, based on observed data, are compared with the results of mathematical calculations by considering the position of the interface between the surface and bottom currents, their velocities, slopes and other phenomena. The two opposite currents appear to be active constantly, although the velocity of the bottom current sometimes is extremely small or almost nonexistent. When this occurs, the water-level decrease in the Bosporus, between the Marmara and Black Seas, should exceed 36cm. According to observations and calculations, such cases are rare exceptions and are limited to brief periods.

Translator

STATIONARY MODEL OF WATER MOVEMENT IN A STRATIFIED STRAIT (USING BOSPORUS [ISTANBUL BOGAZI] AND OTHER STRAITS AS EXAMPLES)

The hydrological regime of southern seas, such as the Black, Mediterranean and Red Seas, is largely determined by water exchange through connective straits. The main properties of these straits are a two-layer structure of currents and a considerable stratification of water, which are determined by differences in the components of fresh water portions of southern seas from surrounding portions of the World Ocean.* Accordingly, it is useful to consider the problem of water movement as part of the system made up of a sea and a strait and affected by certain external factors, such as wind, heat and water balance. However, the solution to such a problem is associated with nearly unsurmountable mathematical difficulties.

This paper discusses a more simplified problem -- namely, the investigation of currents and densities in the Bosporus and other straits, on the condition that differences in densities and water levels at end points and at a given wind over the strait are known. Within such a framework the solution of a number of important practical and scientific problems is possible, notably to evaluate conditions at which the lower level water masses become stratified in straits. The warm and saline water flowing along the bottom from mediterranean seas exercises considerable influence on the biology of the deep water of oceans and inland seas. This process is of great significance in the formation of salinity and biological water structure of the Black Sea.

The hydrodynamic model of water movement in a strait has a number of characteristics. Investigations of steady state currents in straits, which are based on two-layer (Defant, 1961; Tolmazin, 1962) and multi-layer density models (Takano, 1958), are associated with unavoidable deficiencies. Thus, the existence of a certain density stratification and absence of density transport by currents must be assumed. As a result, the processes associated with a /19
diffusive vertical water transport were not investigated. According to K. Takano, the omission of this diffusive vertical water transport has lead to certain discrepancies: the theoretical intensity of integral transport of mediterranean water at Gibraltar differs from the observed value.

In order to eliminate these discrepancies and form a more accurate concept on mechanics of water movement in stratified straits, we discuss the stationary model of currents and density field by means of a nonlinear equation of turbulent diffusion (Tolmozin, 1964).

*This property enables us to consider such straits as being stratified. Zubov (1956) calls them straits with density-induced water exchange.

The main simplifications of our problem (omission of nonlinear members in motion equations and the neglect of vertical advection in diffusion equation) are based on numerical evaluations of the relative significance of these processes in the formation of major fields. These calculations made, for the Bosporus, Dardanelles (Canakkale Bogazi) and other straits by analyzing the Pritchard Calculations (1954, 1956) for stratified estuaries agree with the latter.

You should note that the suggested model is reminiscent of the Rattry and Hansen (1962) model, plotted for a stratified estuary. These writers, however, did not account for one of the important moving forces in a strait — namely, the longitudinal water level drop.

Definition of the Problem on Water Movement in a Stratified Strait

Dynamics of processes occurring in sea straits is determined (1) by the action of known climatic factors on the sea surface, which create the drop of sea levels and densities at the ends of straits, and (2) by the action of wind tension on the water expanse. To find the stationary fields that are formed under these conditions, we use a simplified system of equations of motion and turbulent diffusion:

$$\mu\frac{\partial^2 u}{\partial z^2}=\frac{\partial p}{\partial x}; \qquad (1)$$

$$\frac{\partial p}{\partial z}=g\rho; \qquad (2)$$

$$u\frac{\partial \rho}{\partial x}=k\frac{\partial^2 \rho}{\partial z^2}, \qquad (3)$$

where u = velocity projection on x axis; ρ = density; p = pressure; /20
μ and k = coefficients of turbulent viscosity and diffusion, assumed to be constant; g = acceleration of gravity. The x axis of a rectangular system of coordinates is directed along undisturbed surface to the seaward where the density is smaller; the z axis runs vertically downward. The given strait is assumed to have a rectangular intersection, a constant width and variable depth, h.

We introduce deviation of density ρ' from a certain constant value ρ_*:

$$\rho' = \rho - \rho_*. \tag{4}$$

Eliminating pressure, p, by cross differentiation, we find in lieu of (1)-(3) that:

$$\mu \frac{\partial^2 u}{\partial z^2} = g \frac{\partial \rho'}{\partial x}; \tag{5}$$

$$u \frac{\partial \rho'}{\partial x} = k \frac{\partial^2 \rho'}{\partial z^2}. \tag{6}$$

Equation (5) enables us to introduce an auxiliary function Q determined by equations:

$$u = \frac{1}{\mu} Q_x; \tag{7}$$

$$\rho' = \frac{1}{g} Q_{zzz}. \tag{8}$$

Here the subscripts denote differentiation on the basis of the corresponding variable.

Introducing (7) and (8) into (6), we arrive at the basic equation

$$Q_x Q_{xzzz} = \mu k Q_{zzzzz}. \tag{9}$$

Boundary conditions for the function, Q, can be derived from conditions of velocity and density:

$$\mu \left(\frac{\partial u}{\partial z}\right)_{z=-\zeta} = Q_{xz}(x, -\zeta) = -T; \tag{10}$$

$$u|_{z=h} = Q_x(x, h) = 0; \tag{11}$$

$$\left(\frac{\partial \rho'}{\partial z}\right)_{z=-\zeta} = Q_{zzzz}(x, -\zeta) = 0; \tag{12}$$

$$\rho'|_{z=-\zeta} = \frac{1}{g} Q_{zzz}(x, -\zeta). \tag{13}$$

where ζ = excess water level over the disturbed surface;
T = wind friction against water.

Condition (13) denotes that the density distribution at surface is given. Consider that this change occurs in accordance with the exponential law:

$$\rho'(x, -\zeta) = \rho_n'\left(\frac{x}{l}\right)^{e},$$

where ℓ = length of strait; ρ_n' = const. at surface. The lacking fifth condition with regard to z can be determined from the continuity equation at a given rule of water level change, $\zeta(x)$

$$\frac{d}{dx}\int_{-\zeta}^{h} u\,dz = \frac{d}{dx}\int_{-\zeta}^{h} Q_x\,dz.$$

With a great degree of accuracy, we can write

$$\frac{d}{dx}\int_{0}^{h} Q_x\,dz + \frac{d}{dx}[\zeta Q_x(x, 0)]=0. \qquad (14)$$

Equation (9) requires one condition with regard to x. A given density at one end of the strait ($x = x_1$) in the form of the known function

$$\rho'=\rho'(x_1, z).$$

can be used as such a condition.

We examine the dimensionless variable by assuming that

$$\left.\begin{array}{l} x=l\bar{\xi}, \quad z=H_0\bar{z}, \quad Q=Q_0\bar{Q}, \quad h=H_0\bar{h} \\ \rho'=\rho_0'\bar{\rho}', \quad u=u_0\bar{u}, \quad \zeta=\zeta_0\bar{\zeta} \end{array}\right\}. \qquad (15)$$

where Q_o, H_o, u_o, ρ_o' and ζ_o = characteristic values of the corresponding magnitudes. Accounting for (15), we arrive at the basic equation in a dimensionless form:

$$\varepsilon\bar{Q}_{zzzzz}=\bar{Q}_{\xi}\bar{Q}_{\xi zzz}, \qquad (16)$$

where $\varepsilon = \frac{\mu k l^2}{Q_0 H_0^2}$ = a small dimensionless parameter.

Due to the smallness of ζ, the surface conditions can be transferred to $z = 0$:

$$\left.\begin{array}{l} \bar{Q}_{\xi\bar{z}}(\xi, 0) = \frac{\tau}{\lambda}, \quad \bar{Q}_{\xi}(\xi, \bar{h})=0 \\ Q_{\bar{z}\bar{z}\bar{z}\bar{z}}(\xi, 0)=0, \quad Q_{\bar{z}\bar{z}\bar{z}}(\xi, 0)=\gamma\xi^{n} \\ \frac{d}{d\xi}\int_{0}^{h}\bar{Q}_{\xi}d\bar{z} + \frac{\zeta_0}{H_0}\cdot\frac{u_0}{\Delta U_0}\cdot\frac{d}{d\xi}[\bar{\zeta}\bar{Q}_{\xi}(\xi, 0)]=0 \end{array}\right\}. \qquad (17)$$

where $\tau = -\frac{TH_0}{u_0\mu}$, $\gamma = \frac{g\rho'_u H_0^3}{Q_0}$, $\lambda = \frac{g\rho' H_0^2}{lT}$, ΔU_0 = a typical change of the mean vertical speed.

The typical scale of Q can be expressed by known magnitudes, depending on the predominant current component, which may be either a wind or density component. In case the wind-induced currents, as defined in (17) predominate,

$$Q_0 = lH_0T. \qquad (18)$$

can be used.

If the currents are determined mainly by a longitudinal density drop, we have

$$Q_0 = g\rho'_0 H_0^3. \qquad (19)$$

Because the main current system of straits is created by nonmechanical causes, expression (19) should be used in the evaluation of Q_0. Thus,

$$\varepsilon = \frac{\mu k l^2}{g\rho'_0 H_0^5}.$$

Assume that $H_0 = 7.2 \cdot 10^3$cm; $\ell - 3 \cdot 10^6$cm; $\rho'_0 = 10^{-2}$ г$\cdot$cm^{-3}; $\mu = 300$ г $\cdot$cm^{-1} $\cdot$ cek^{-1}; K = 5.3cm $\cdot$cek^{-2} (conditions in the Bosporus). Under these conditions $\varepsilon = 0.763 \cdot 10^{-4}$. For other straits, where H_0 is larger, ε is still smaller (Table 1).

The finite magnitudes of the problem (typical depth, velocity, etc.) have been closer with the design that the size of the area and the magnitude of the nonuniform member were near unity. The dominant behavior of $\bar{Q}$ in the equation appeared to be a small parameter at the leading derivative. This fact enables us to formulate our problem as that of a boundary layer. On the basis of such an approach, the character of the vertical profile, $\bar{Q}$, can be studied without solving the equation (16).

The concept "boundary layer," which usually has been associated with a phenomenon occurring near a body when the magnitudes of Reynold's numbers are large, is now associated with many aspects of applied mechanics. Such a generalization is discussed by G. F. Carrier (Kerrier) (1955). The boundary layer denotes a zone where the given function increases at a rapid rate; therefore, at a certain distance from the boundary the value of the function /23

TABLE 1

Parameter	Bosporus	Bab-el-Mandeb	Gibraltar
h_1, м	48	150	350
h_2, м	72	350	—
H_0, м	72	400	500
l, км	30	80*	100**
ξ_1	0,861	0,225	1,094
ξ_2	1,861	1,225	2,094
α	—0,525	—0,500	—0,550
m	0,722	0,791	0,666
μ, г/см sec	300	200	150
k, см/sec.	5,3	64	100
$\varepsilon \cdot 10^4$	0,762	0,707	0,050
ρ_0, г/см^3	1,0080	1,0225	0,0265
u_0, см/sec	100	100	100
ΔU, см/sec	5,0	0,3	0,5
ζ_0, см	10	3	1
$\lambda \cdot 10^2$	0,41	3,33	8,33

NOTE:

*Distance from Cape Raheita to the deep portion of the Gulf of Aden.

**Distance from the east end of strait to the deep portion of the Atlantic Ocean.

becomes "smooth" (inner area). The main property of the boundary layer, according to Carrier, is the small parameter at the leading derivative in the main equation. /23

In our case, the existence of boundary layers is possible near the surface and bottom. Let us appraise the thickness of these layers (areas of sharp increase of function $\bar{Q}$). For this purpose, we change the scale of coordinates that are normal to the boundaries. Because ζ is very small, we assume that

$$\chi_1 = \varepsilon^n \bar{z}, \tag{20}$$

for bottom ($\bar{h} = \varphi(\xi)$ = known function

$$\chi_2 = \varepsilon^n [\bar{z} - \varphi(\xi)].$$

The n value in the coordinates of boundary layers is selected so that the order with respect to ε in the left part of (16) coincides with the order with respect to ε in the corresponding part of $Q_\xi Q_{\xi zzz}$. Introducing these coordinates into (16), we can assume that $n = -1/3$. Thus, the thickness of the boundary layer has the order of $\varepsilon l/3H_0$.

By introducing the typical values ε and H_0, calculated for various straits, the thickness of boundary layer appears to be nearly equal to 10m.

There is no need to discuss separately the boundary layer and the inner area because a special solution to (16) is necessary. We note that the neglect of a member with parameter leads to degeneration of equation (16). This is evidently associated with the character of motion in stratified straits: in a two-layer current, the velocity u (or Q_ξ) becomes 0. Evidently, in the center of the inner area we have to deal with small left and right-hand parts of equation (16).

The qualitative analysis enables us to describe the behavior of function $\bar{Q}$ in vertical direction: a sharp increase of function Q at the bottom to a certain maximum is replaced by a zone where the value of the function decreases uniformly and the $\bar{Q}$ reaches 0, then increases again, with another sign, at the surface. Thus, the

small parameter ε determines the appearance of boundary layers when defining the process with the aid of supplementary function Q. The thickness of boundary layers (their wash-out) depends, by means of ε, upon the value of viscosity and diffusion coefficients. /24

Automatic Model Solution to the Main Equation

The suggested method for solving (16) and (17) has been successfully used in a number of projects dealing with ocean circulation. The main feature of it is the transformation of coordinates at which the equation can be reduced to a simple form. The solution of this equation is a function whose profile has the property of similarity for all intersections of the area, differing only in the scale.

$\bar{Q}$ can be expressed as follows:

$$\bar{Q}=\varepsilon M(\xi)f(\eta), \qquad (21)$$

where

$$\eta=\bar{z}\xi^{\alpha}m^{-1}.$$

Here α and m are unknown parameters determined by the geometric shape of the given strait. To solve $0\leq\bar{h}\leq 1$, it is necessary to define the pattern of change in the depth of the given strait:

$$\bar{h}=m\xi^{-\alpha}. \qquad (22)$$

It is evident that the actual values of $\bar{h}$ and η will be materialized when ξ varies within $\xi_1\leq\xi\leq\xi_2$, where ξ_1 and ξ_2 are dimensionless coordinates of the beginning and the end of the strait, whereby $\xi_1>0$. The variation of parameters α, m and ξ, enables us to determine quite accurately the general slope of bottom in a number of straits.

The small parameter of factor ε in (20) does not mean that $\bar{Q}$ is a small value. Here the magnitude scale has been artificially changed (order f equals $\frac{1}{\varepsilon}$) in order to eliminate ε in the equation defining f.

One can readily see that one of the variables can be excluded from (16) only when

$$M=\xi^{2\alpha+2}. \qquad (23)$$

Accounting for (23) and introducing (20) into (16), we have the following simple equation for f:

$$f^{V}=m^2\alpha^2\eta^2 f^{IV} f^{I}+a\eta f^{IV} f+b\eta f^{III} f^{I}+c f^{III} f. \qquad (24)$$

The Roman numbers denote in this case the order of differentiation with respect to η.

Boundary conditions for f become /25

$$\frac{f^{I}(0)}{\varepsilon}=\frac{m\tau}{\varepsilon^{3}\lambda(3\alpha+2)\xi^{3\alpha+1}}=D_1=\text{const}; \qquad (25)$$

$$f^{I}(1)+sf(1)=0; \qquad (26)$$

$$f^{IV}(0)=0; \qquad (27)$$

$$\frac{f^{III}(0)}{\varepsilon}=\frac{m^3\rho_n'\xi^{\theta}}{\varepsilon^2\rho_0'\xi^{5\alpha+2}}=D_2=\text{const}; \qquad (28)$$

$$\int_0^1[sf+\eta f^{I}]\,d\eta+s\varkappa f(0)=0. \qquad (29)$$

In (24)—(29) the following designations are introduced:

$$a=m^2\alpha(2\alpha+2);\quad b=m^2\alpha(5\alpha+2);\quad c=m^2(2\alpha+2)(5\alpha+2);$$

$$s=\frac{2\alpha+2}{\alpha};\quad \varkappa=\frac{u_0 n\zeta_0}{\Delta U_0 m H_0}.$$

In order to preserve the given character of the process, it is necessary to impose certain restrictions on the longitudinal distribution of density at surface, and tangential wind pressure and surface:

$$\tau=\tau_0\xi^{3\alpha+2},\quad \rho'(\xi_1,\,0)=\rho_n'\xi^{5\alpha+2},\quad \zeta=n\xi^{-\alpha}, \qquad (30)$$

Here n = proportionality coefficient determined by means of ξ values at the end of a given strait.

It follows from (29) that $\varkappa=0$ is applicable when the loss of water in the given strait equals zero (losses of surface and bottom currents are equal).

Let us plot solution for (24)-(29), by expanding f(η) in series of ε degrees. Assume

$$f=\varepsilon\sum_{n=0}^{\infty}\varepsilon^n f_n. \qquad (31)$$

By introducing (31) into (24) and equalizing the coefficients at equal powers of ε, we arrive at the following system of equations:

$$f_0^{V}=0;$$
$$f_1^{V}=m^2\varkappa^2\eta^2 f_0^{I} f_0^{IV}+a\eta f_0^{IV} f_0+b\eta f_0^{I} f^{III}+c f_0 f^{III};$$
$$f_2^{V}=m^2\varkappa^2\eta^2 (f_0^{I} f_1^{IV}+f_1^{I} f_0^{IV})+a\eta (f_0 f_1^{IV}+f_1 f_0^{IV})+$$
$$+b\eta (f_0^{I} f_1^{III}+f_1^{I} f_0^{III})+c (f_0 f_1^{III}+f_1 f_0^{III})$$

Accounting for the boundary conditions, we find a solution for the zero and first approximations: /26

$$f_0=A_0+\sum_{i=1}^{4} A_i \frac{\eta^i}{i!}; \tag{32}$$

$$f_1=B_0+A_2 \sum_{i=1}^{8} B_i \frac{\eta^i}{i!}, \tag{33}$$

where

$$A_0=\frac{1}{24s(3\varkappa+2)}[(s+3)D_2-12(s+1)D_1];$$
$$A^1=D_1;$$
$$A_2=-\frac{1}{4(s+2)(3\varkappa+2)}[(s+3)(4\varkappa+3)D_2+(s+1)(2\varkappa+1)D_1];$$
$$A_3=D_2;$$
$$B_0=\frac{A_3}{s(3\varkappa+2)}\sum_{i=5}^{8} A_{i-5}\frac{[(i-5)b+c](s+i)(i-2)}{(i+1)!};$$
$$B_2=-\frac{6A_0}{(s+2)(3\varkappa+2)}\sum_{i=5}^{8} A_{i-5}\frac{[(i-5)b+c][i(\varkappa+1)+\varkappa](s+i)}{(i+1)!};$$
$$B_{i+5}=(ib+c)A_i. \quad A_4=B_1=B_3=B_4=0.$$

The subsequent approximations in which high powers of ε occurs, do not introduce substantial variations to the distribution of density and velocity.

The Effect of External Factors on Vertical Distribution of Density and Velocity in a Strait

To calculate the vertical velocity profile and density distribution with the first approximation ($f_0+\varepsilon f_1$), we assume that $\alpha = -0.525$; $m = 0.722$; $\varepsilon 0.763\cdot 10^{-4}$.

Figure 1 shows the vertical velocity profiles in windless periods. The velocity values of surface and bottom currents vary proportionally to the density gradient in longitudinal direction.

The effect of water level decrease on current velocity is shown in Fig. 2 (the other factors are assumed to be constant). With an increase in longitudinal slope between seas directed against density gradient, the velocity of the upper current and the depth of interface increase, while the velocity of the lower current decreases. /27

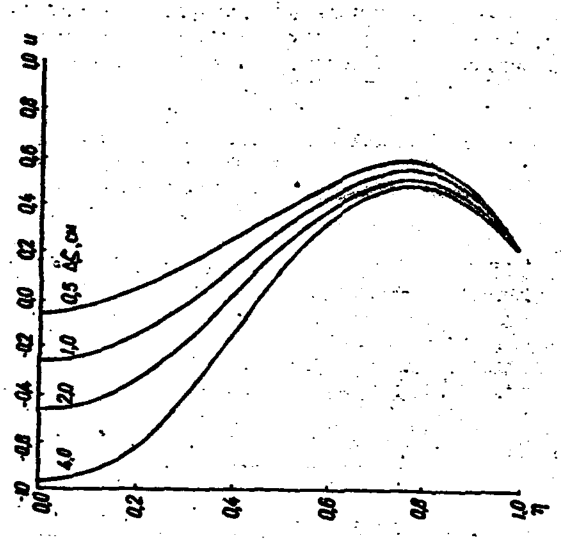

FIG. 2. The Effect of Water Level Decrease on Vertical Profile of Current Velocity in Perfectly Calm Weather and at Constant D_2.

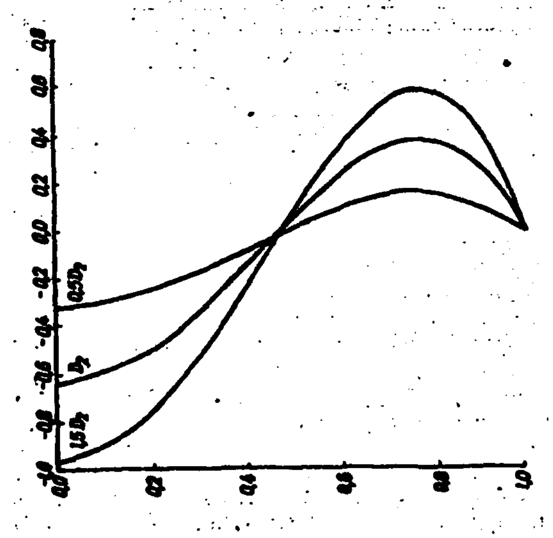

FIG. 1. The Effect of Density Decrease, D_2, on the Vertical Profile of Current in Perfectly Calm Weather (D_2=4.31·10^7; $\varkappa$=0.1; ω=0.0 m/sec).

Winds exercise a somewhat different influence on the vertical velocity profile (Fig. 3). With a wind blowing against the direction of density decrease, the velocity of the upper current increases. Simultaneously, the velocity of the lower current increases (as with an increase in the density gradient). This characteristic is associated with the assumed stationary nature of the process: preservation of the constancy of volume in seas at various D_2 and W and fixed $\Delta\zeta$ must be associated with a definite relationship between the losses in both the upper and lower currents. As seen from Fig. 3, the velocity of wind-induced currents is limited.

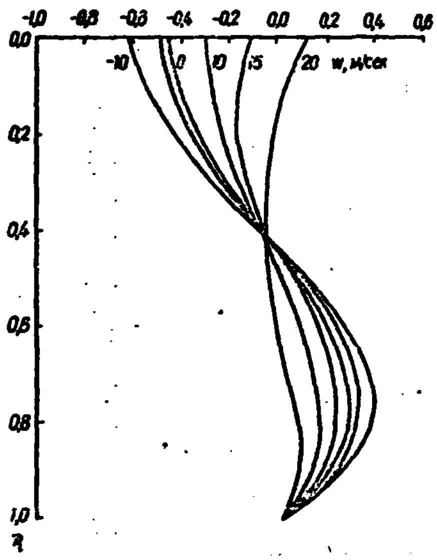

FIG. 3. The Effect of Longitudinal Wind Component on Velocity Profile at a Constant Sea Level Decrease ($\Delta\zeta$ = 3cm) and Density Gradient (sign minus denotes that the wind blows in the direction of surface current).

The greatest variations in current regime are usually associated with wind-induced de-levelling of free surface in pre-tidal sea areas. This phenomenon, based on observations, is supported by formulae derived in this paper. Indeed, the wind action in straits is smaller than gradient forces, but the density difference changes little, mainly due to seasonal variations in the component of thermal and salinity balance of the seas. Therefore, the velocity values and thickness of water layers, involved by the upper and bottom currents, depend mainly on the slope of sea level in the given strait.

The formation of a three-layer structure of currents is possible with winds blowing against the slope of free surface (see Fig. 3, ω = 20 m/sec). Such a phenomenon has been observed several times in Bab-el-Mandeb Strait when the currents near the surface or bottom have the same sign and countercurrent flows in the intermediate layer (Muromtsev, 1960; Thompson, 1939).

Two approximations lead to the following formula of density deviation:

$$\bar{\rho}' = \varepsilon^2 m^{-2\xi(5\varkappa+2)} D_2 \Big\{ 1 + \frac{m^3 \varepsilon D_2 (5\varkappa+2)}{24(3\varkappa+2)} \times \left[\frac{1}{2}\eta^3 - \frac{4\varkappa+3}{4}\eta^4 + \frac{3\varkappa+2}{5}\eta^5 \right] - \frac{m^2 \varepsilon D_1 (5\alpha+2)(3\alpha+2)}{2(3\varkappa+2)} \times \left[\frac{1}{2}\eta^2 - \frac{3\varkappa+2}{3}\eta^3 + \frac{2\varkappa+1}{4}\eta^4 \right]. \qquad (35)$$

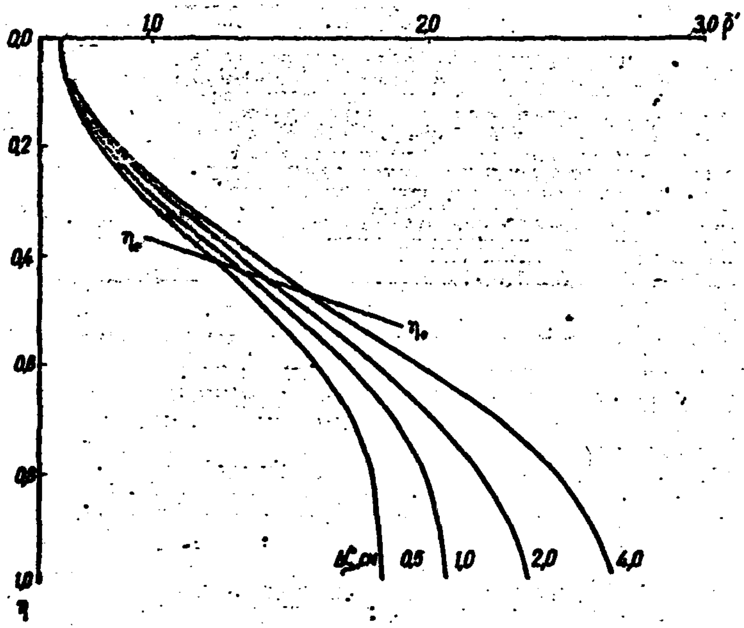

FIG. 4. The Effect of Water Level Decrease on Vertical Distribution of Density Deviation When $\omega=0$ and D_2=const. (line η_*-η_* shows the depth of interface between water masses).

Figure 4 illustrates the vertical distribution of density deviation. With an increase in the slope of the surface directed against the density gradient ($\varkappa \leq 0$), the interface between water masses lowers. The depth of this surface for various $\varkappa$ is shown in Fig. 4 by line η_*-η_*. In addition, the degree of water stratification increases

with an increase in the slope of water level. Noteworthy is the fact that wind affects stratification in a similar way, but it does not lead to a lowering of interface. /30

The amplitude and character of the influence exercised by various factors (wind, longitudinal density gradient, and water level slope) on the vertical current field and density correspond to our concepts on these phenomena, formed by actual observations.

Application of Automatic Model Solutions to Concrete Conditions in Bosporus and Other Straits

The solution discussed in this paper enables us to investigate the problem not in a general form but with special designations of functions $\rho'(\xi)$ and $\bar{h}(\xi)$. With the aid of a known law of density distribution at water surface and the depth values taken from bathymetric charts describing the ends of a given strait it is possible to determine five unknown parameters of the problem, namely: D_2, ξ_1, ξ_2, m and α. For this purpose, one has to solve the following system of equations:

$$\left.\begin{array}{l} \bar{\rho}'(\xi_1, 0) = s^2 m^{-3} \xi_1^{5\alpha+2} D_2 \\ \bar{\rho}'(\xi_2, 0) = s^2 m^{-3} \xi_2^{5\alpha+2} D_2 \\ \bar{h}_1 = m \xi_1^{-\alpha} \\ \bar{h}_2 = m \xi_2^{-\alpha} \\ \xi_2 - \xi_1 = 1 \end{array}\right\} . \qquad (36)$$

The first two equations of (36) are based on (28) defining two density values at the surface of the ends of a given strait. The next two are based on (22). The last equation denotes that a dimensionless length of strait equals 1. Thus, in order to find the unknown parameters, it is necessary to know the mean depth at the end of straits and have data on density values at the ends of the strait for the meteorological situation associated with the given problem. In practice, the determination of the general slope of the bottom for most of the straits and the selection of extreme depth values appear to be most difficult.

Merz (1928) and V. L. Lebedev (Zubov, 1956) determined the areas of transverse cross sections in various places of Bosporus and calculated the mean depths. These calculations enabled us to determine the values of $\bar{h}_1$ and $\bar{h}_2$. Analogous magnitudes for Bab-el-Mandeb Strait could be determined only approximately by using Sea

Atlas (1953). It was assumed that the north end of the strait lies over a threshold near Cape Raheita (see Table 1).

The complex bottom relief of the Strait of Gibraltar obstructed selection of the general bottom slope. Therefore, in lieu of determining α and m with the aid of bottom profile, these parameters /31
were determined by means of relationships between density deviations at bottom and surface and the mean depth at one end of the strait. These relationships are based on observations and written as follows:

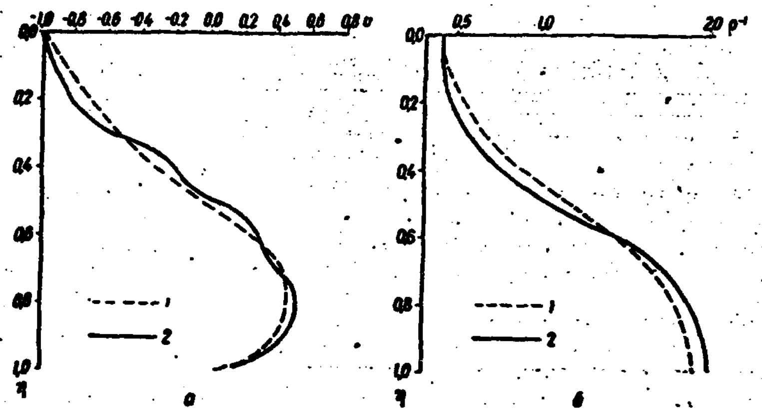

Рис. 5. Сопоставление рассчитанных (1) и наблюденных (2) (Merz, 1928) скоростей (а) и отклонений плотности (б) для пролива Босфор.

FIG. 5. Comparison of Calculated (1) and Observed (2) (Merz, 1928) Velocities (a) and Deviations of Density (б) for Bosporus.

Fig. 5 demonstrates the relation between observed and calculated values of current velocities and vertical curves of density deviations for the Bosporus. The number of such cases for various straits where calculations correspond to observed data can be increased.

With the aid of the calculation, the wedging of water masses in a strait, which results from diffusion processes, can be found. For this purpose, one must find the interface between the surface and bottom water. It is known that the penetration of saline water of

the bottom current in front of the strait depends upon the intensity of mixing taking place in the strait. The interface between two water masses in a strait (h_*) is usually associated with an isopycnic plane in longitudinal cross section (e.g., $\rho = 1.020$ g/cm^3 in the Bosporus). By using (35), the position of the surface of cross section for each strait can be calculated. Then the depth drop of this surface can be calculated with the aid of the length of the strait Δh_*. In Table 2 are presented the calculated and observed magnitudes of Δh_* by making use of the isopycnic plane. /32

TABLE 2

Strait	Depth Drop	
	Calculated	Observed
Bosporus $\rho = 1.020$	33	36.5
Bab-el-Mandeb $\rho = 1.0265$	127	110* 160**
Gibraltar $\rho = 1.028$	379	420*

NOTE:
*Data by Schott, 1942
**Data by G. Neumann and McGill

Observations conducted in straits have made it possible to determine that the boundary between water masses has a greater slope than the boundary between currents. This incongruence probably depends on differences between diffusion flow and the quantity of motion. Theoretical calculations confirm it. The depth of zero speed, h_c, is determined by vertical profiles of current velocities, which have been calculated for several points in a given strait. According to calculations under the specified conditions, Δh_c (depth h_c decrease in the strait) is 12m in the Bosporus, 52^m in Bab-el-Mandeb and 22m in Gibraltar. The observed difference between Δh_c in the Bosporus is 18m (Marz, 1928). For other straits we do not have such data. According to assumption by Defant (1961), Δh_c for Bab-el-Mandeb is 40m, for Gibraltar 15m.

The noted difference between calculated and observed magnitudes of Δh_* and Δh_c probably result from our assumption that coefficients of viscosity and diffusion are constant. In the last analysis, the position of boundaries h_* and h_c depends on diffusion flow and quantity of motion, respectively. Each of these processes depends, in turn, upon the gradient of the corresponding property in vertical direction and the magnitude of coefficients. Our assumption that coefficients μ and k are constant introduces errors into the values of density and current gradients, which are superimposed over the errors inherent in the coefficients themselves. Despite this disadvantage, the h_* and h_c values (calculated and observed) differ little. Therefore, the suggested theory appears to be useful.

The resultant solutions enable us to clarify a very important practical question: under what conditions is the lower current

absent in straits? Ulliot and Ilgaz (1946) doubted that the lower Bosporus Current is constant. Before them R. de Buen (1929) proved that the influx of mediterranean water into the /33 Atlantic Ocean is an episodic phenomenon.

By using the Bosporus as an example, let us find out under what conditions the bottom current ceases to exist in the strait. Under average conditions, the water level decrease in the 30km stretch of the strait is 6cm. This situation corresponds to our problem that $\varkappa = 0.05$. Calculations demonstrate that with $\varkappa = 0.3$, the bottom current has a very small velocity. Consequently, in order to eliminate the two-level system of currents in the Bosporus, the sea level decrease between the Black and Marmara Seas must exceed 36cm. Direct observations have demonstrated (Smith, 1946) that such a slope seldom occurs, and it does not represent a steady state. This confirms the concept of many oceanographers (Vodyanitskiy, 1948; Bogdanova, 1959; Pektas, 1959) namely, that the influx of Marmara Sea water into the Black Sea is continuous.

BIBLIOGRAPHY

1. BOGDANOVA, A. K. VODOOBMEN CHEREZ BOSFOR I YEGO ROL' V PEREMESHIVANII VOD CHERNOGO MORYA (Water Exchange Through Bosporus and Its Significance in the Mixing of the Black Sea Water). In TRUDY SEVASTOPOL'SKOY BIOLOGICHESKOY STANTSII, Vol. XII, M., 1959.

2. VODYANITSKIY, V. A. OSNOVNOY VODOOBMEN I ISTORIYA FORMIROVANIYA SOLENOSTI CHERNOGO MORYA (The Main Problem and History of Salinity Formation in the Black Sea). In TRUDY SEVASTOPOL'SKOY BIOLOGICHESKOY STANTSII, Vol. VI, M., 1948.

3. ZUBOV, N. N. OSNOVY UCHENIYA O PROLIVAKH MIROVOGO OKEANA (Fundamentals in the Science of Straits in the World Ocean). M., 1956.

4. KERRIER, G. F. ZADACHI POGRANICHNOGO SLOYA V PRIKLADNOY MEKHANIKE (Problems of Boundary Layers in Mechanics). In PROBLEMY MEKHANIKI, IL, M. 1955.

5. MORSKOY ATLAS (Sea Atlas). IZD. GLAVNOGO SHTABA VOYENNO-MORSKIKH SIL, Vol. I, 1953.

6. MUROMTSEV, A. M. K GIDROLOGII KRASNOGO MORYA (Hydrology of the Caspian Sea). DAN SSSR, Vol. 134, No. 6, 1960.

7. TOLMAZIN, D. M. O POLE TECHENIY I VODOOBMENE V PROLIVE BOSFOR (Current Field and Water Exchange in Bosporus). OKEANOLOGIYA, Vol. 2, 1, 1962.

8. TOLMAZIN, D. M. K TEORII TECHENIY V MORSKIKH PROLIVAKH (Theory of Currents in Sea Straits). DAN SSSR, Vol. 159, No. 1, 1964.

9. DeBUEN, R. FENOMENOS DINAMICOS EN ESTRECH DE GIBRALTAR (Dynamic Phenomenon in the Strait of Gibraltar). CONGRESSO INST. DE OCEANOGR., Sevilla, 1929.

10. DEFANT, A. Physical Oceanography, Vol. 1, Pergamon Press, 1961.

11. MERZ, A. HYDROGRAPHISCHER BEOBACHTUNGEN IN BOSPORUS UND DARDANELLEN (Hydrographic Observations in Bosporus and Dardanelles). VEROFF. MEERESKUNDE, N. F., Berlin 1, 1929.

12. NEUMANN, G., McGILL, D. A. Circulation of Red Sea in Early Summer. DEEP-SEA RES., Vol. 8, No. 3/4, 1961.

13. PEKTAS, H. The Influence of the Mediterranean Water on the Hydrography of the Black Sea. Fisheries Center, Istambul. Technical Papers, 4-th Meeting, 1956.

14. PRITCHARD, D. W. A Study of the Salt Balance in a Coastal Plain Estuary. J. MARINE RES., Vol. 13, NI, 1954.

15. PRITCHARD, D. W. The Dynamic Structure of a Coastal Plain Estuary. J. MARINE RES., Vol. 15, No. 3, 1956.

16. RATTRY, M., HANSEN, D. JR. A Similarity Solution for Circulation in a Coastal Plain Estuary. J. OF MARINE RES., Vol. 20, 1962.

18. SCHOTT, G. GEOGRAPHIE DES ATLANTISCHEN OZEANS (Geography of the Atlantic Ocean). Hamburg, 1942.

19. SMITH, W. E. Some Observations on Water Levels and Other Phenomena Along the Bosphorus. TRANS. AMER. GEOPHYS. UNION, Vol. 27, 1, 1946. /34

20. TAKANO, K. ETUDE SUR L'EXTENSION DES CAUX MEDITERRANEENNES DANS L'OCEANS ATLANTIQUE (Study of the Extension of the Mediterranean Sea into the Atlantic Ocean). 1 part - BULL. INST. OCEANOGR., No. 1124, 1958.

21. TOMPSON, E. F. The Exchange of Water Between the Red Sea and the Gulf of Aden Over the "Sill". J. Murray Exped., 1933. SCI. REPTS., Vol. 2, 4, 1939.

22. ULLIOT, P., ILGAZ, O. The Hydrography of the Bosporus. THE AMER. GEOGRAPHICAL SOCIETY OF N. Y., Vol. VI, No. 2, 1946.

23. VERCELLI, F. COMPAGHA IDOGRAFIC NEL MAREE ROSSO "AMMIRAGLIO MAGNAGHI" 1923/24 (Sea Currents). RICERCHE DI OCEANOGRAFIA FISICA, Part 1a, Correnti e moree.

ooOoo

Security Classification

DOCUMENT CONTROL DATA - R & D			
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)			
1. ORIGINATING ACTIVITY (Corporate author) U. S. Naval Oceanographic Office Washington, D. C. 20390		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP	
3. REPORT TITLE "Stationary Model of Water Movement in a Stratified Strait (Using Bosporus [Istanbul Boğazi] and Other Straits as Examples)" (STATSIONARNAYA MODEL' DVIZHENIYA VOD V STRATIFITSIROVANNOM PROLIVE (V PRIMENENII K BOSFORU I NEKOTORYN DRUGIM PROLIVAM)			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Translation from Russian			
5. AUTHOR(S) (First name, middle initial, last name) D. M. TOLMAZIN			
6. REPORT DATE	7a. TOTAL NO. OF PAGES 20		7b. NO. OF REFS 23
8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NOO TRANS 501 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)		
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.			
11. SUPPLEMENTARY NOTES INSTITUT BIOLOGII YUZHNYKH MOREY, Vol. 20, No. 2, Pp. k8-34, KIEV 1967	12. SPONSORING MILITARY ACTIVITY U. S. Naval Oceanographic Office Washington, D. C. 20390		

13. ABSTRACT

Two-layer water stratification in straits, such as the Bosporus, Bab-el-Mandeb, and Gibraltar, is discussed. Conclusions, based on observed data, are compared with the results of mathematical calculations by considering the position of the interface between the surface and bottom currents, their velocities, slopes and other phenomena. The two opposite currents appear to be active constantly, although the velocity of the bottom current sometimes is extremely small or almost nonexistent. When this occurs, the water-level decrease in the Bosporus, between the Marmara and Black Seas, should exceed 36cm. According to observations and calculations, such cases are rare exceptions and are limited to brief periods.

TRANSLATOR

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Straits						
2. Stratification						
3. Hydrodynamics						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification